UNSTEADY HEAT CONDUCTION OF A THIN ROD WITH A UNIFORMLY MOVING FUSION BOUNDARY OR A THERMALLY INSULATED BOUNDARY

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A solution is obtained to the problem of unsteady heat conduction of a semi-infinite thin rod, thermally insulated along its generators, or a plate, with a uniformly moving fusion boundary, or a thermally insulated boundary. The results are presented of a numerical calculation for a copper rod with various rates of motion of the boundary.

We consider a semi-infinite thin rod thermally insulated along its generators, a finite part, of length a, being free from thermal insulation and having thermal contact with the surrounding medium.

The following two conditions will be examined at the moving end boundary:

1. The end boundary is fused by a moving heat source proceeding along the rod with velocity u. The fused material is carried away.

2. Removal of rod material from the end proceeds with velocity u, and there are no heat fluxes at the end.

At the same time, the thermal insulation is broken down at the same rate, so that the length of the uninsulated part remains constant and equal to a. The heat fluxes along a generator have intensity q, constant with time. The last condition exists, for example, in the case when the part of the rod stripped of thermal insulation is surrounded by a high-temperature medium, while heat transfer is accomplished by radiation according to the Stefan-Boltzmann law.

The heat conduction equations,

$$\frac{\partial T_1}{\partial \tau} = a_1^2 \frac{\partial^2 T_1}{\partial x^2},$$

$$\frac{\partial T_2}{\partial \tau} = a_1^2 \frac{\partial^2 T_2}{\partial x^2} + \frac{2qa_1^2}{R\lambda},$$

may be written as follows in the system of coordinates $\xi = x - u\tau$ moving along with the fusion boundary:

$$\frac{\partial T_1}{\partial \tau} = a_1^2 \frac{\partial^2 T_1}{\partial \xi^2} + u\frac{\partial T_1}{\partial \xi}, \quad a<\xi<\infty, \quad \tau>0; \tag{1}$$

$$\frac{\partial T_2}{\partial \tau} = a_1^2 \frac{\partial^2 T_2}{\partial \xi^2} + u\frac{\partial T_2}{\partial \xi} + \frac{2qa_1^2}{R\lambda},$$

$$0<\xi<a, \quad \tau>0; \tag{2}$$

$$T_1(\xi, \tau)|_{\tau=0} = T_2(\xi, \tau)|_{\tau=0} = 0; \tag{3}$$

$$T_1(\xi, \tau)|_{\xi=\infty} = 0, \quad T_2(\xi, \tau)|_{\xi=0} = T_f - T_0; \tag{4}$$

$$T_1(\xi, \tau)|_{\xi=a} = T_2(\xi, \tau)|_{\xi=a},$$

$$\left.\frac{\partial T_1(\xi, \tau)}{\partial \xi}\right|_{\xi=a} = \left.\frac{\partial T_2(\xi, \tau)}{\partial \xi}\right|_{\xi=a}. \tag{5}$$

It is assumed that at any cross section the temperature difference $\Delta T_2(\xi, \tau)$ between the surface and the axis of the thin rod is negligible in comparison with the temperature $T_2(\xi, \tau)$ at the section.

The degree of accuracy of this assumption may be evaluated from the ratio $\frac{\Delta T_2}{T_2} \approx \frac{qR}{\lambda T_2}$, which is easily obtained from the heat balance equation on the rod surface.

Applying a Laplace transformation with respect to the variable τ to the system (1), (2), and the boundary and contact conditions (4), (5), we obtain

$$\frac{d^2\bar{T}_1(\xi, s)}{d\xi^2} + \frac{u}{a_1^2}\frac{d\bar{T}_1(\xi, s)}{d\xi} - \frac{s\bar{T}_1(\xi, s)}{a_1^2} = 0, \tag{6}$$

$$\frac{d^2\bar{T}_2(\xi, s)}{d\xi^2} + \frac{u}{a_1^2}\frac{d\bar{T}_2(\xi, s)}{d\xi} - \frac{s\bar{T}_2(\xi, s)}{a_1^2} = -\frac{2q}{sR\lambda}, \tag{7}$$

$$\bar{T}_1(\xi, s)|_{\xi=\infty} = 0, \quad \bar{T}_2(\xi, s)|_{\xi=0} = (T_f - T_0)/s, \tag{8}$$

$$\bar{T}_1(\xi, s)|_{\xi=a} = \bar{T}_2(\xi, s)|_{\xi=a},$$

$$\left.\frac{d\bar{T}_1(\xi, s)}{d\xi}\right|_{\xi=a} = \left.\frac{d\bar{T}_2(\xi, s)}{d\xi}\right|_{\xi=a}. \tag{9}$$

The system (6), (7) has the following solution:

$$\bar{T}_1(\xi, s) = A\exp\left(-\frac{u\xi}{2a_1^2} - \frac{\xi}{a_1}\sqrt{\frac{u^2}{4a_1^2} + s}\right) + B\exp\left(-\frac{u\xi}{2a_1^2} + \frac{\xi}{a_1}\sqrt{\frac{u^2}{4a_1^2} + s}\right), \tag{10}$$

$$\bar{T}_2(\xi, s) = C\exp\left(-\frac{u\xi}{2a_1^2} - \frac{\xi}{a_2}\sqrt{\frac{u^2}{4a_1^2} + s}\right) + D\exp\left(-\frac{u\xi}{2a_1^2} + \frac{\xi}{a_1}\sqrt{\frac{u^2}{4a_1^2} + s}\right) + \frac{2qa_1^2}{s^2R_1\lambda}. \tag{11}$$

It follows from boundary conditions (8) that B = 0. Satisfying the boundary and contact conditions, we obtain

$$\bar{T}_1(\xi, s) = \bar{F}_1(\xi, s)\frac{1}{s} + \frac{T_f - T_0}{s} \times \exp\left(-\frac{u\xi}{2a_1^2}\right)\exp\left[-\frac{1}{a_1}\sqrt{\frac{u^2}{4a_1^2} + s\xi}\right], \tag{12}$$

$$\bar{T}_2(\xi, s) = \bar{F}_2(\xi, s)\frac{1}{s} + \frac{T_f - T_0}{s} \times \exp\left(-\frac{u\xi}{2a_1^2}\right)\exp \times \left[-\frac{1}{a_1}\sqrt{\frac{u^2}{4a_1^2} + s\xi}\right] + \frac{2qa_1^2}{s^2R\lambda}, \tag{13}$$

where

$$\bar F_1(\xi,s)=-\frac{qua_1}{2R\lambda}\exp\left[\frac{u}{2a_1^2}(a-\xi)\right]\times$$
$$\times\exp\left[-\frac{1}{a_1}\sqrt{\frac{u^2}{4a_1^2}+s(\xi-a)}\right]\Big/s\sqrt{\frac{u^2}{4a_1^2}+s}+$$
$$+\frac{qua_1}{2R\lambda}\exp\left[\frac{u}{2a_1^2}(a-\xi)\right]\times$$
$$\times\exp\left[-\frac{1}{a_1}\sqrt{\frac{u^2}{4a_1^2}+s(a+\xi)}\right]\Big/s\sqrt{\frac{u^2}{4a_1^2}+s}+$$
$$+\frac{qa_1^2}{R\lambda}\exp\left[\frac{u}{2a_1^2}(a-\xi)\right]\exp\left[-\right.$$
$$\left.-\frac{1}{a_1}\sqrt{\frac{u^2}{4a_1^2}+s(\xi-a)}\right]\Big/s+$$
$$+\frac{qa_1^2}{R\lambda}\exp\left[\frac{u}{2a_1^2}(a-\xi)\right]\exp\left[-\right.$$
$$\left.-\frac{1}{a_1}\sqrt{\frac{u^2}{4a_1^2}+s(a+\xi)}\right]\Big/s-$$
$$-\frac{2qa_1^2}{R\lambda}\exp\left(-\frac{u\xi}{2a_1^2}\right)\exp\left[-\frac{1}{a_1}\sqrt{\frac{u^2}{4a_1^2}+s\xi}\right]\Big/s,$$

$$\bar F_2(\xi,s)=-\frac{qua_1}{2R\lambda}=\exp\left[\frac{u}{2a_1^2}(a-\xi)\right]\times$$
$$\times\exp\left[-\frac{1}{a_1}\sqrt{\frac{u^2}{4a_1^2}+s(a-\xi)}\right]\Big/s\sqrt{\frac{u^2}{4a_1^2}+s}+$$
$$+\frac{qua_1}{2R\lambda}\exp\left[\frac{u}{2a_1^2}(a-\xi)\right]\times$$
$$\times\exp\left[-\frac{1}{a_1}\sqrt{\frac{u^2}{4a_1^2}+s(a+\xi)}\right]\Big/s\sqrt{\frac{u^2}{4a_1^2}+s}-$$
$$-\frac{qa_1^2}{R\lambda}\exp\left[\frac{u}{2a_1^2}(a-\xi)\right]\exp\left[-\right.$$
$$\left.-\frac{1}{a_1}\sqrt{\frac{u^2}{4a_1^2}+s(a-\xi)}\right]\Big/s+$$
$$+\frac{qa_1^2}{R\lambda}\exp\left[\frac{u}{2a_1^2}(a-\xi)\right]\exp\left[-\right.$$
$$\left.-\frac{1}{a_1}\sqrt{\frac{u^2}{4a_1^2}+s(a+\xi)}\right]\Big/s-$$
$$-\frac{2qa_1^2}{R\lambda}\exp\left(-\frac{u\xi}{2a_1^2}\right)\exp\left[-\frac{1}{a_1}\sqrt{\frac{u^2}{4a_1^2}+s\xi}\right]\Big/s.$$

The transforms $\bar F_1(\xi,s)$ and $\bar F_2(\xi,s)$ have the following originals [1], respectively:

$$F_1(\xi,\Theta)=\frac{qa_1^2}{R\lambda}\left\{\mathrm{erf}\left(\frac{\xi}{2a_1\sqrt{\Theta}}+\frac{u\sqrt{\Theta}}{2a_1}\right)-\right.$$
$$-\mathrm{erf}\left(\frac{\xi-a}{2a_1\sqrt{\Theta}}+\frac{u\sqrt{\Theta}}{2a_1}\right)+$$
$$+\exp\left(-\frac{u\xi}{a_1^2}\right)\left[\mathrm{erf}\left(\frac{\xi}{2a_1\sqrt{\Theta}}-\right.\right.$$
$$\left.\left.\left.-\frac{u\sqrt{\Theta}}{2a_1}\right)-\mathrm{erf}\left(\frac{a+\xi}{2a_1\sqrt{\Theta}}-\frac{u\sqrt{\Theta}}{2a_1}\right)\right]\right\},\qquad(14)$$

$$F_2(\xi,\Theta)=\frac{qa_1^2}{R\lambda}\left\{\mathrm{erf}\left(\frac{\xi}{2a_1\sqrt{\Theta}}+\frac{u\sqrt{\Theta}}{2a_1}\right)+\right.$$
$$+\mathrm{erf}\left(\frac{a-\xi}{2a_1\sqrt{\Theta}}-\frac{u\sqrt{\Theta}}{2a_1}\right)-2+$$
$$+\exp\left(-\frac{u\xi}{a_1^2}\right)\left[\mathrm{erf}\left(\frac{\xi}{2a_1\sqrt{\Theta}}-\frac{u\sqrt{\Theta}}{2a_1}\right)-\right.$$
$$\left.\left.-\mathrm{erf}\left(\frac{a+\xi}{2a_1\sqrt{\Theta}}-\frac{u\sqrt{\Theta}}{2a_1}\right)\right]\right\}.\qquad(15)$$

The originals of the expressions $2qa_1^2/s^2R\lambda$ and $([T_f - T_0]/s)\exp(-u\xi/2a_1^2)\exp[(-1/a_1)\times\sqrt{u^2/4a_1^2+s\xi}]$, respectively, will be

$$\frac{2qa_1^2}{R\lambda}\tau,\qquad(16)$$

and

$$\frac{T_f-T_0}{2}\left[\exp\left(-\frac{u\xi}{a_1^2}\right)\mathrm{erfc}\left(\frac{\xi}{2a_1\sqrt{\tau}}-\frac{u\sqrt{\tau}}{2a_1}\right)+\right.$$
$$\left.+\mathrm{erfc}\left(\frac{\xi}{2a_1\sqrt{\tau}}+\frac{u\sqrt{\tau}}{2a_1}\right)\right].\qquad(17)$$

The Laplace transformation has the following property: if there is an original $f_1(\tau)$ for the transform $\bar F(s)$, then for $\bar F(s)/s$ the original may be written in the form $f_2(\tau)=\int_0^\tau f_1(\Theta)d\Theta$ [1].

Applying this to the transforms (12) and (13), and using expressions (16) and (17), we obtain, after some transformations, the solution of the original equations (1) and (2):

$$T_1(\xi,\tau)=\frac{qa_1^2}{R\lambda}\int_0^\tau\left\{\mathrm{erf}\left(\frac{\xi}{2a_1\sqrt{\Theta}}+\right.\right.$$
$$\left.+\frac{u\sqrt{\Theta}}{2a_1}\right)-\mathrm{erf}\left(\frac{\xi-a}{2a_1\sqrt{\Theta}}+\frac{u\sqrt{\Theta}}{2a_1}\right)+$$
$$+\exp\left(-\frac{u\xi}{a_1^2}\right)\left[\mathrm{erf}\left(\frac{\xi}{2a_1\sqrt{\Theta}}-\frac{u\sqrt{\Theta}}{2a_1}\right)-\right.$$
$$\left.\left.-\mathrm{erf}\left(\frac{a+\xi}{2a_1\sqrt{\Theta}}-\frac{u\sqrt{\Theta}}{2a_1}\right)\right]\right\}d\Theta+$$
$$+\frac{T_f-T_0}{2}\exp\left(-\frac{u\xi}{a_1^2}\right)\times$$
$$\times\mathrm{erfc}\left(\frac{\xi}{2a_1\sqrt{\tau}}-\frac{u\sqrt{\tau}}{2a_1}\right)+$$
$$+\frac{T_f-T_0}{2}\mathrm{erfc}\left(\frac{\xi}{2a_1\sqrt{\tau}}+\frac{u\sqrt{\tau}}{2a_1}\right),\qquad(18)$$

$$T_2(\xi,\tau)=\frac{qa_1^2}{R\lambda}\int_0^\tau\left\{\operatorname{erf}\left(\frac{\xi}{2a_1\sqrt{\Theta}}+\frac{u\sqrt{\Theta}}{2a_1}\right)+\right.$$
$$+\operatorname{erf}\left(\frac{a-\xi}{2a_1\sqrt{\Theta}}-\frac{u\sqrt{\Theta}}{2a_1}\right)+$$
$$+\exp\left(-\frac{u\xi}{a_1^2}\right)\left[\operatorname{erf}\left(\frac{\xi}{2a_1\sqrt{\Theta}}-\frac{u\sqrt{\Theta}}{2a_1}\right)-\right.$$
$$\left.\left.-\operatorname{erf}\left(\frac{a+\xi}{2a_1\sqrt{\Theta}}-\frac{u\sqrt{\Theta}}{2a_1}\right)\right]\right\}d\Theta+$$
$$+\frac{T_f-T_0}{2}\exp\left(-\frac{u\xi}{a_1^2}\right)\times$$
$$\times\operatorname{erfc}\left(\frac{\xi}{2a_1\sqrt{\tau}}-\frac{u\sqrt{\tau}}{2a_1}\right)+$$
$$+\frac{T_f-T_0}{2}\operatorname{erfc}\left(\frac{\xi}{2a_1\sqrt{\tau}}+\frac{u\sqrt{\tau}}{2a_1}\right). \qquad (19)$$

If the initial temperature is different from zero, then for $T_1(\xi,\tau)$ and $T_2(\xi,\tau)$ we must understand $T_1(\xi,\tau)-T_0$ and $T_2(\xi,\tau)-T_0$, respectively. It is not hard to verify that $T_1(\xi,\tau)$ and $T_2(\xi,\tau)$ satisfy the initial (3), boundary (4), and contact (8) conditions.

For strictness, we must verify whether or not $T_1(\xi,\tau)$ and $T_2(\xi,\tau)$ are solutions of the original equations (1) and (2).

Substituting (18) into the appropriate original equation (1), we obtain

$$\frac{\sqrt{\pi}}{2}\left\{\operatorname{erf}\left(\frac{\xi}{2a_1\sqrt{\tau}}+\frac{u\sqrt{\tau}}{2a_1}\right)-\operatorname{erf}\left(\frac{\xi-a}{2a_1\sqrt{\tau}}+\frac{u\sqrt{\tau}}{2a_1}\right)+\right.$$
$$+\exp\left(-\frac{u\xi}{a_1^2}\right)\left[\operatorname{erf}\left(\frac{\xi}{2a_1\sqrt{\tau}}-\frac{u\sqrt{\tau}}{2a_1}\right)-\right.$$
$$\left.\left.-\operatorname{erf}\left(\frac{a+\xi}{2a_1\sqrt{\tau}}-\frac{u\sqrt{\tau}}{2a_1}\right)\right]\right\}-$$
$$-\frac{u}{2a_1}\int_0^\tau\frac{1}{\sqrt{\Theta}}\left\{\exp\left[-\left(\frac{\xi}{2a_1\sqrt{\Theta}}+\frac{u\sqrt{\Theta}}{2a_1}\right)^2\right]-\right.$$
$$\left.-\exp\left[-\left(\frac{\xi-a}{2a_1\sqrt{\Theta}}+\frac{u\sqrt{\Theta}}{2a_1}\right)^2\right]\right\}d\Theta+$$
$$+\frac{1}{2}\int_0^\tau\frac{1}{\Theta}\left\{\exp\left[-\left(\frac{\xi}{2a_1\sqrt{\Theta}}+\right.\right.\right.$$
$$\left.\left.+\frac{u\sqrt{\Theta}}{2a_1}\right)^2\right]\left(\frac{\xi}{2a_1\sqrt{\Theta}}+\frac{u\sqrt{\Theta}}{2a_1}\right)-$$
$$-\exp\left[-\left(\frac{\xi-a}{2a_1\sqrt{\Theta}}+\frac{u\sqrt{\Theta}}{2a_1}\right)^2\right]\left(\frac{\xi-a}{2a_1\sqrt{\Theta}}+\right.$$
$$\left.\left.+\frac{u\sqrt{\Theta}}{2a_1}\right)\right\}d\Theta-\exp\left(-\frac{u\xi}{a_1^2}\right)\cdot$$
$$\cdot\frac{1}{2}\int_0^\tau\frac{1}{\Theta}\left\{\exp\left[-\left(\frac{a+\xi}{2a_1\sqrt{\Theta}}-\frac{u\sqrt{\Theta}}{2a_1}\right)^2\right]\times\right.$$
$$\times\left(\frac{a+\xi}{2a_1\sqrt{\Theta}}-\frac{u\sqrt{\Theta}}{2a_1}\right)-\exp\left[-\left(\frac{\xi}{2a_1\sqrt{\Theta}}-\frac{u\sqrt{\Theta}}{2a_1}\right)^2\right]\times$$
$$\left.\times\left(\frac{\xi}{2a_1\sqrt{\Theta}}-\frac{u\sqrt{\Theta}}{2a_1}\right)\right\}d\Theta+\exp\left(-\frac{u\xi}{a_1^2}\right)\times$$
$$\times\frac{u}{2a_1}\int_0^\tau\frac{1}{\sqrt{\Theta}}\left\{\exp\left[-\left(\frac{\xi}{2a_1\sqrt{\Theta}}-\frac{u\sqrt{\Theta}}{2a_1}\right)^2\right]-\right.$$
$$\left.-\exp\left[-\left(\frac{a+\xi}{2a_1\sqrt{\Theta}}-\frac{u\sqrt{\Theta}}{2a_1}\right)^2\right]\right\}d\Theta=0. \qquad (20)$$

If the solution $T_1(\xi,\tau)$ obtained satisfies the corresponding equation (1), then the equality (20) must be fulfilled identically for any $\tau>0$, $\xi>0$.

Denoting the left part in (20) by $\Phi(\xi,\tau)$, and differentiating it with respect to τ, we shall check the existence of the identity

$$\frac{\partial\Phi(\xi,\tau)}{\partial\tau}\equiv 0,\quad \text{i. e.}\quad \Phi(\xi,\tau)\equiv f(\xi). \qquad (21)$$

The identity (21) must be satisfied for any $\tau\gg 0$, $\xi>a$. But for any $\xi>a$ in (21) we have [see (20)]

$$\lim_{\tau\to 0}\Phi(\xi,\tau)=0,$$

i.e., expression (21) may be written in the form

$$\Phi(\xi,\tau)\equiv 0.$$

Therefore, the expression obtained, (18), satisfies the corresponding original equation (1). Proceeding similarly, it may be verified that $T_2(\xi,\tau)$ is a solution of (2).

The results of the numerical calculation of expressions (18) and (19) for a copper rod cooled along a generator are presented in Fig. 1.

If removal of material proceeds from the end of the rod, while there is no heat flux through the moving end boundary, then the equations of heat conduction in the system of coordinates $\xi=x-u\tau$, moving along with the thermally insulated boundary, are written as follows:

$$\frac{\partial T_1}{\partial\tau}=a_1^2\frac{\partial^2T_1}{\partial\xi^2}+u\frac{\partial T_1}{\partial\xi},\quad \infty>\xi>a,\quad \tau>0; \qquad (22)$$

$$\frac{\partial T_2}{\partial\tau}=a_1^2\frac{\partial^2T_2}{\partial\xi^2}+u\frac{\partial T_2}{\partial\xi}+\frac{2qa_1^2}{R\lambda},$$
$$a>\xi>0,\quad \tau>0; \qquad (23)$$

$$T_1(\xi,\tau)|_{\tau=0}=T_2(\xi,\tau)|_{\tau=0}=0; \qquad (24)$$

$$T_1(\xi,\tau)|_{\xi=\infty}=0,\quad \left.\frac{\partial T_2(\xi,\tau)}{\partial\xi}\right|_{\xi=0}=0; \qquad (25)$$

$$T_1(\xi,\tau)|_{\xi=a}=T_2(\xi,\tau)|_{\xi=a},$$

$$\left.\frac{\partial T_1(\xi,\tau)}{\partial\xi}\right|_{\xi=a}=\left.\frac{\partial T_2(\xi,\tau)}{\partial\xi}\right|_{\xi=a}. \qquad (26)$$

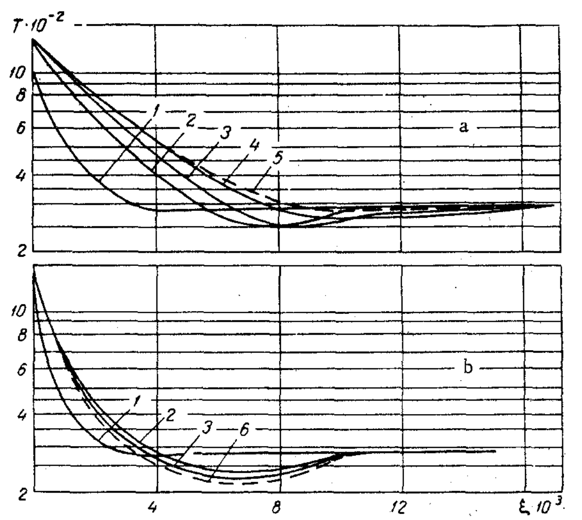

Fig. 1. Temperature of the rod with a) $u = 2 \cdot 10^{-2}$ and b) $8 \cdot 10^{-2}$ m/sec: 1) for $\tau = 0.01$ sec; 2) 0.05; 3) 0.1; 4) 0.3; 5, 6) steady temperature profiles with τ respectively 1.0 and 0.3 sec (T in °K, λ in m).

Applying a Laplace transformation with respect to the variable τ to the system (22), (23), and to the boundary and contact conditions (25), (26), we obtain for the corresponding transforms the expressions

$$\frac{d^2\bar{T}_1(\xi, s)}{d\xi^2} + \frac{u}{a_1^2}\frac{d\bar{T}_1(\xi, \tau)}{d\xi} - \frac{s\bar{T}_1(\xi, s)}{a_1^2} = 0, \quad (27)$$

$$\frac{d^2\bar{T}_2(\xi, s)}{d\xi^2} + \frac{u}{a_1^2}\frac{d\bar{T}_2(\xi, s)}{d\xi} - \frac{s\bar{T}_2(\xi, s)}{a_1^2} = -\frac{2q}{sR\lambda}, \quad (28)$$

$$\bar{T}_1(\xi, s)|_{\xi=\infty} = 0, \quad \left.\frac{d\bar{T}_2(\xi, s)}{d\xi}\right|_{\xi=0} = 0, \quad (29)$$

$$\bar{T}_1(\xi, s)|_{\xi=a} = \bar{T}_2(\xi, s)|_{\xi=a},$$

$$\left.\frac{d\bar{T}_1(\xi, s)}{d\xi}\right|_{\xi=a} = \left.\frac{d\bar{T}_2(\xi, s)}{d\xi}\right|_{\xi=a}. \quad (30)$$

The method of solution of the equations obtained is similar to that examined above.

The final expressions for temperature $T_1(\xi, \tau)$ and $T_2(\xi, \tau)$ have the following form:

$$T_1(\xi, \tau) = \frac{qa_1^2}{R\lambda}\int_0^\tau \left[\operatorname{erfc}\left(\frac{\xi - a}{2a_1\sqrt{\Theta}} + \frac{u\sqrt{\Theta}}{2a_1}\right) - \right.$$

$$\left. - \exp\left(-\frac{ua}{a_1^2}\right)\operatorname{erfc}\left(\frac{\xi + a}{2a_1\sqrt{\Theta}} + \frac{u\sqrt{\Theta}}{2a_1}\right)\right]d\Theta, \quad (31)$$

$$T_2(\xi, \tau) = -\frac{qa_1^2}{R\lambda}\int_0^\tau \left[\operatorname{erfc}\left(\frac{a - \xi}{2a_1\sqrt{\Theta}} - \frac{u\sqrt{\Theta}}{2a_1}\right) + \right.$$

$$\left. + \exp\left(-\frac{ua}{a_1^2}\right)\operatorname{erfc}\left(\frac{\xi + a}{2a_1\sqrt{\Theta}} + \frac{u\sqrt{\Theta}}{2a_1}\right)\right]d\Theta + \frac{2qa_1^2}{R\lambda}\tau. \quad (32)$$

If the initial rod temperature is different from zero, then for $T_1(\xi, \tau)$ and $T_2(\xi, \tau)$ we must understand $T_1(\xi, \tau) - T_0$ and $T_2(\xi, \tau) - T_0$, respectively.

It is easy to verify that $T_1(\xi, \tau)$ and $(T_2(\xi, \tau)$ satisfy the original equations (22), (23), and the initial (24), boundary (25), and contact (26) conditions.

The results of numerical calculation for a copper rod heated along a generator are presented in Fig. 2. Solutions for similar problems with a plate may be obtained by replacing the rod radius R by the plate thickness δ in expressions (18), (19), (31), and (32). Numerical calculations were carried out for the following original data: $R = 0.5 \cdot 10^{-3}$m, $T_0 = 293°$ K, $a = 10^{-2}$m, $u = 2 \cdot 10^{-2}$ and $8 \cdot 10^{-2}$ m/sec, $a_1^2 = 0.97 \cdot 10^{-4}$ m²/sec, $T_f = 1353°$ K, $\lambda = 3.6 \cdot 10^2$ W/m · °K, $q = \mp 80 \cdot 10^4$ W/m².

If the heat fluxes on the rod are realized by convection with heat transfer coefficient $\alpha = 4.0 \cdot 10^2$ W/m²·°K, and if the temperature T_∞ of the surrounding medium is 2273 ° K, then, assuming negligible variation with time of the temperature drop $T_\infty - T_2(\xi, \tau)$, it may be assumed in (31) and (32) that $q = \alpha\,[T_\infty - T_2(\xi, \tau)] \approx \alpha(T_\infty - T_0) \approx 80.0 \cdot 10^4$ W/m².

The greatest deviation of the quantity $T_\infty - T_2(\xi, \tau)$ from the initial temperature drop $T_\infty - T_0$, as may be seen from Fig. 2, is 5% and 20% at velocities of $8 \cdot 10^{-2}$ and $2 \cdot 10^{-2}$ m/sec, respectively.

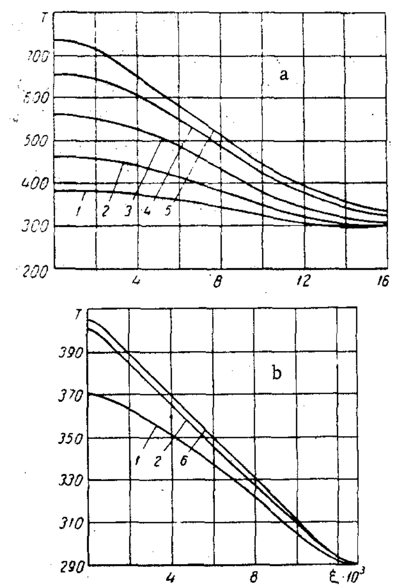

Fig. 2. Temperature of the rod with a) $u = 2 \cdot 10^{-2}$ and b) $8 \cdot 10^{-2}$ m/sec: 1) for $\tau = 0.1$ sec; 2) 0.2; 3) 0.4; 4) 0.8; 5, 6) steady temperature profiles with τ of 2.0 and 0.4 sec, respectively.

NOTATION

$T_1(\xi, \tau)$—temperature of thermally insulated part of rod; $T_2(\xi, \tau)$—temperature of rod at section stripped of thermal insulation; q—heat flux on unit lateral surface of rod; a_1^2—thermal diffusivity; λ—thermal conductivity; α—coefficient of heat transfer from surrounding medium to rod; T_0—initial temperature of rod; T_∞—temperature of surrounding medium; R—radius of rod; a—length of part of rod free from thermal insulation; u—rate of displacement of boundary; T_f—fusion temperature of rod.

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